Lec 9

(1)

$V = -\int_{-}^{+}$ من ال - لل +

Capacitors

* Electrical device has 2 conductors, sperated Through dielectric medium, store equal & opposite charge

- $V_1 - V_2 = -\int_{V_2}^{1} \bar{E} \cdot d\bar{l}$

- $C = \dfrac{Q \text{ of one plate}}{V \text{ pot dif}} = \dfrac{\oint \bar{E} \cdot d\bar{s}}{\int \bar{E} \cdot d\bar{l}}$ Farad

Cap. of Parallel plate cap → cylindrical cap → spherical

① Parallel plate cap

$z = d$: + +

$E_2 = \frac{\rho}{2\epsilon_0}(-\hat{a_z})$, $E_1 = \frac{\rho}{2\epsilon_0}(-\hat{a_z})$

$z = 0$: −

$E_1 = E_2 = \dfrac{-\rho_s}{2\epsilon}(\hat{a_z})$.

$E_t = E_1 + E_2 = -\dfrac{\rho_s}{\epsilon_0} \hat{a_z}$

$\rho_s = \dfrac{Q}{A}$, $\therefore E_t = \dfrac{-Q}{\epsilon A} \hat{a_z}$

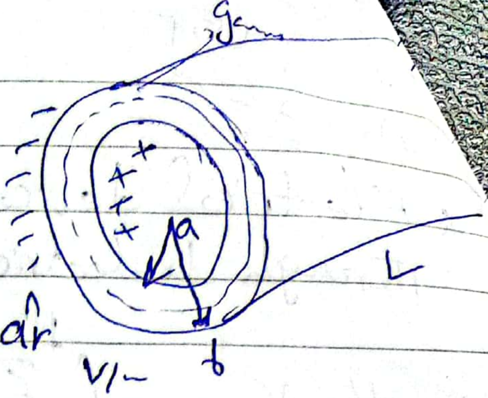
$V = -\int_2^1 \bar{E} \cdot d\bar{l} = -\int_{z=0}^{(z=d)} \dfrac{-Q}{\epsilon A} (a_z) \cdot dz \hat{a_z}$

$$\boxed{V = \frac{Q}{\epsilon A}[z]_0^d = \frac{Qd}{\epsilon A}}$$

$$\boxed{C = \frac{Q}{V} = \frac{Q \epsilon A}{Q d} \therefore \frac{\epsilon A}{d} = \frac{\epsilon_0 \epsilon_r A}{d}}$$

Energy stored $= \frac{1}{2}\int \epsilon E^2 dv = \begin{cases} = \dfrac{Q^2}{2C} \\ = \dfrac{1}{2} QV \\ = \dfrac{1}{2} CV^2 \end{cases}$

(2) Coaxial Cap



$\rho_L = Q/L$

$$E = \frac{\rho_L}{2\pi\epsilon r}\hat{a_r} = \frac{Q}{2\pi\epsilon r L}\hat{a_r}$$

$$V = -\int_b^a E \cdot \bar{dl} = -\int_b^a \bar{E}\, dr\, \hat{a_r} \qquad \bar{dl} = dr\,\hat{a_r}$$

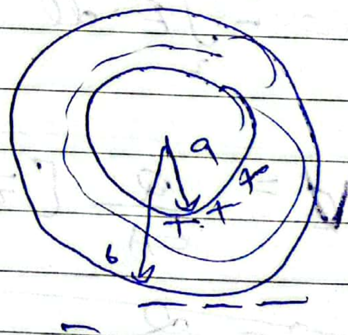
$$= -\int_b^a \frac{Q}{2\pi\epsilon r L}\, dr$$

$$= -\frac{Q}{2\pi\epsilon L}\left[\ln r\right]_b^a$$

$$V = \frac{Q}{2\pi\epsilon L}\left[\ln b - \ln a\right] = \frac{Q}{2\pi\epsilon L}\ln\frac{b}{a}$$

$$\boxed{C = \frac{Q}{V} = \frac{2\pi\epsilon L}{\ln(b/a)}}$$

(3) Spherical Cap



$$Q = \epsilon\oint \bar{E}\cdot d\bar{s} \qquad E = \frac{Q}{4\pi\epsilon r^2}\hat{a_r}$$

$$V = -\int \bar{E}\cdot\bar{dl} = -\int_b^a \frac{Q}{4\pi\epsilon r^2}\cdot dr \qquad \bar{dl} = dr\,\hat{a_r}$$

$$= -\frac{Q}{4\pi\epsilon}\left[\frac{r^{-1}}{-1}\right]_b^a = \frac{Q}{4\pi\epsilon}\left[\frac{1}{r}\right]_b^a$$

$$V = \frac{Q}{4\pi\epsilon}\left[\frac{1}{a} - \frac{1}{b}\right] = \frac{Q}{4\pi\epsilon}\,\frac{b-a}{ab}$$

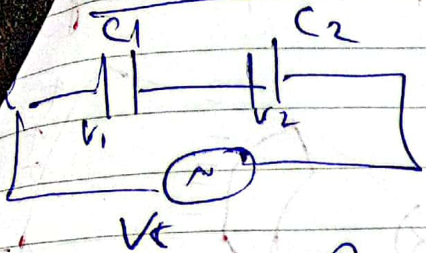
$$C = \frac{Q}{V} = \frac{4\pi\epsilon ab}{b-a}$$

Note $\epsilon = \frac{k}{r^2\left(\frac{b-a}{ab}\right)}$

ions

3

series Connection



I ثابت Q

$$Q_t = Q_1 = Q_2$$

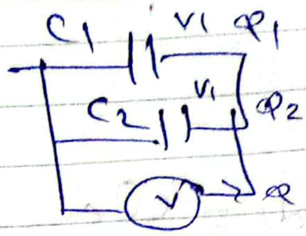
$$\cancel{C V_t = C_1 V_1 = Q_2 V_2}$$

$$V_t = V_1 + V_2$$

$$\frac{Q}{C_t} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{eq}}$$

parallel Connection



$V_1 = V_2 = V$

$$Q_t = Q_1 + Q_2$$

$$\boxed{Q = CV}$$

$$\cancel{C V_t = C_1 V_1 + C_2 V_2}$$

$$V_t = V_1 = V_2$$

$$\boxed{C_{eq} = C_1 + C_2}$$

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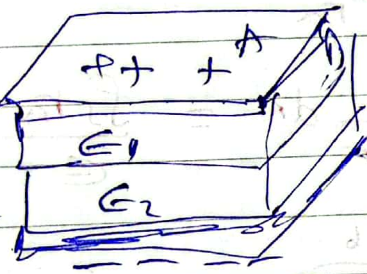
{ توالي / توازي stacked dielectric cap.

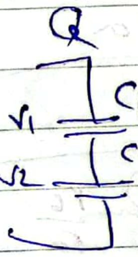
$$V = V_1 + V_2$$



$$Q_1 = Q_2 = Q$$

$$C_1 = \frac{\epsilon_0 \epsilon_{r_1} A}{d/2} = \frac{Q}{V_1}$$

$$C_2 = \frac{\epsilon_0 \epsilon_{r_2} A}{d/2} = \frac{Q}{V_2}$$

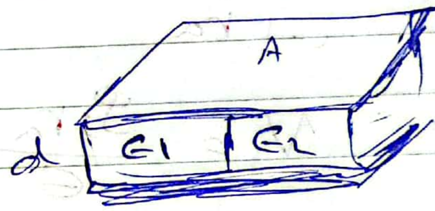


$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{Q}{V}$$

$$V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}$$

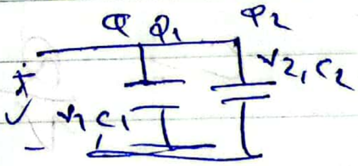
$$E_1 = \frac{V_1}{d_1} \qquad E_2 = V_2/d_2$$

$$W = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{Q^2}{2C}$$



$$V = V_1 + V_2$$

$$Q = Q_1 + Q_2$$



$$C_1 = \frac{\epsilon_0 \epsilon_{r_1} (A/2)}{d}$$

$$C_2 = \frac{\epsilon_0 \epsilon_{r_2} (A/2)}{d}$$

$$C_{eq} = C_1 + C_2 = Q/V$$

$$Q_1 = C_1 V, \quad Q_2 = C_2 V$$

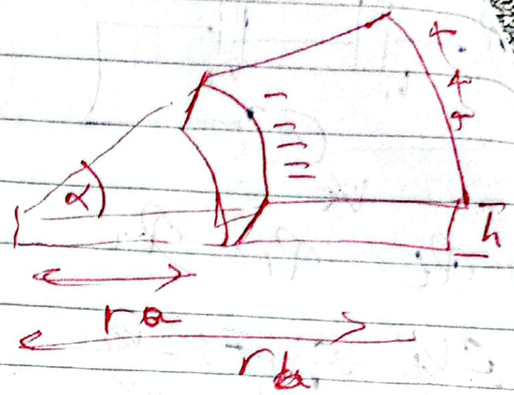
$$E_1 = E_2 = V/d$$

$$W_E = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{Q^2}{2C}$$

EX(1) (4)

find capacitance bet curved plats of shown fig

Sol



$$\oint\!\!\oint \bar{D} \cdot ds = Q_{en}$$

$$D\,[r\,\alpha\,h] = \rho_s\,[r_a\,\alpha\,h]$$

قوس

$$D = \rho_s \frac{r_a}{r}(-\hat{a}_r)$$

$$E = -\frac{\rho_s r_a}{\epsilon r}\hat{a}_r$$

$$V = -\int_{(-)\,r_a}^{(+)\,r_b} E \cdot dl = \int_{r_a}^{r_b} \frac{\rho_s r_a}{\epsilon r}\,dr$$

$$V = \frac{\rho_s r_a}{\epsilon}\int_{r_a}^{r_b}\frac{1}{r}\,dr = \frac{\rho_s r_a}{\epsilon}\ln r\Big]_{r_a}^{r_b}$$

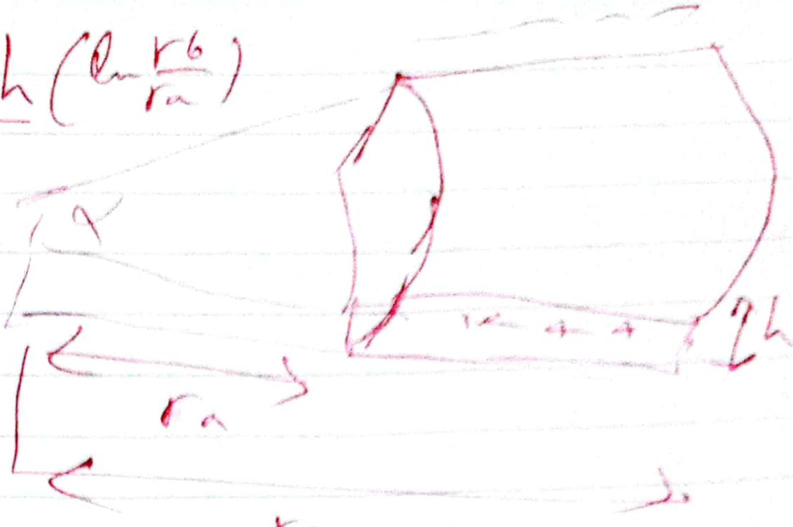
$$V = \frac{\rho_s r_a}{\epsilon}\ln\frac{r_b}{r_a}$$

$$C = \frac{Q}{V} = \frac{Q\epsilon}{\rho_s r_a \ln\frac{r_b}{r_a}} = \frac{\cancel{\rho_s r_a}\,\alpha h \epsilon}{\cancel{\rho_s r_a}\ln\frac{r_b}{r_a}}$$

$$\boxed{C = \frac{\epsilon\alpha h}{\ln\left(\frac{r_b}{r_a}\right)}}\quad \text{Farad}$$

EX(2)

Prove $C = \frac{\varepsilon_0 \varepsilon_r h \left(\ln \frac{r_b}{r_a}\right)}{\alpha}$



Sol

$$V = -\int_{-}^{+} E \cdot dl$$

− التغير العكسي

$$V_0 = -\int_{\alpha}^{0} \bar{E}\, \hat{a_\phi} \underbrace{[r\, d\phi\, \hat{a_\phi}]}_{}^{dl} = -\int_{\alpha}^{0} \bar{E}\, r\, d\phi = E_\phi\, r\, \alpha$$

$$E_\phi = \frac{V_0}{r\alpha}$$

$$D = \epsilon E_\phi = \frac{\epsilon V_0}{\alpha r}$$

$$Q = \iint \bar{D} \cdot \bar{ds} = \iint \frac{\epsilon V_0}{\alpha r} \boxed{ds} \xrightarrow{} h * dr$$

$$= \frac{\epsilon V_0 h}{\alpha} \int_{r_a}^{r_b} \frac{1}{r}\, dr = \frac{\epsilon V_0 h}{\alpha} \ln r \Big|_{r_a}^{r_b}$$

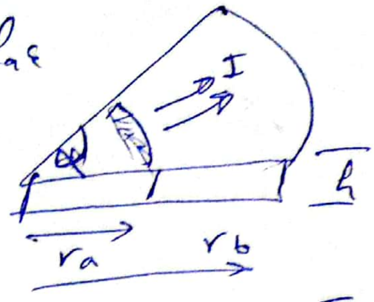
$$Q = \frac{\epsilon V_0 h}{\alpha} \ln\left(\frac{r_b}{r_a}\right)$$

$$C = \frac{Q}{V} = \frac{\epsilon h}{\alpha} \ln \frac{r_b}{r_a}$$

$\epsilon \to \varepsilon_0 \varepsilon_r$

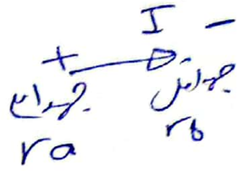
Cap. Lec 10 field

Calc. Resistance between 2 Curvature surface



$$J = \frac{I}{A} = \frac{I}{\alpha r h}$$

$$E = \frac{J}{\sigma} = \frac{I}{\sigma \alpha r h} \hat{a_r}$$



$$V_{ab} = -\int_{r_b}^{r_a} E \cdot dl = -\int_{r_b}^{r_a} \frac{I}{\sigma \alpha r h} dr$$

$$V_{ab} = +\frac{I}{\sigma \alpha h} \int_{r_a}^{r_b} \frac{1}{r} dr = \frac{I}{\alpha \sigma h} \ln r \Big|_{r_a}^{r_b} = \frac{I}{\alpha \sigma h} \ln \frac{r_b}{r_a}$$

$$R = \frac{V_{ab}}{I} = \frac{1}{\alpha \sigma h} \ln \frac{r_b}{r_a}$$

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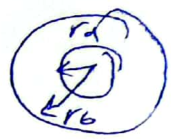
2) 2 Concentric sphere , r_a, r_b



$$\therefore J = \frac{I}{A} = \frac{I}{4\pi r^2}$$

$$E = \frac{J}{\sigma} = \frac{I}{4\pi \sigma r^2} \hat{a_r}$$

$$V_{ab} = -\int_{r_b}^{r_a} E \cdot dl = -\int_{r_b}^{r_a} \frac{I}{4\pi \sigma r^2} dr = \frac{I}{4\pi \sigma} \left(\frac{1}{r}\right)\Big|_{r_b}^{r_a}$$

$$V_{ab} = \frac{I}{4\pi \sigma} \left[\frac{1}{r_a} - \frac{1}{r_b}\right] \quad , \quad R = \frac{V_{ab}}{I} = \frac{1}{4\pi \sigma} \left(\frac{1}{a} - \frac{1}{b}\right)$$

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3) $E = -\nabla V$